

FROM PAIRWISE RELATIONSHIPS TO HIGHER-ORDER NETWORKS: THE EVOLUTION OF GRAPH-BASED MODELLING FOR COMPLEX SYSTEMS

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ABSTRACT

Graph-based modelling has become a general language for representing complex relational systems, yet the meaning of a relationship has changed substantially across the history of the field. Classical graph theory formalized connectivity through vertices and pairwise edges; network science subsequently used these structures to study centrality, clustering, communities, small-world organization, heterogeneous degree distributions and dynamical processes. The increasing availability of relational data, however, exposed situations in which a system cannot be faithfully described as a collection of independent dyadic interactions. Multilayer and temporal networks extended graph representations by preserving relational context and temporal order, while hypergraphs and simplicial complexes introduced explicit representations of interactions involving more than two entities. These representational changes have consequences for diffusion, contagion, synchronization, consensus, cooperation, topology, signal processing and machine learning. This review synthesizes the evolution of graph-based modelling through a hybrid systematic-narrative framework supported by bibliometric methods. Rather than treating higher-order models as replacements for graphs, the paper argues that they constitute a family of representations selected according to the structure of the underlying phenomenon and the inferential question. The review develops an Evolutionary Higher-Order Network Intelligence Framework (EHNIF), linking interaction, representation, structure, dynamics, signals, learning and intelligence. It further identifies unresolved challenges concerning representation selection, empirical ground truth, hypergraph–simplicial distinctions, dynamic higher-order systems, scalability, benchmarking, explainability and causality. The paper concludes with a research agenda oriented toward empirically validated, interoperable, scalable and explainable higher-order network intelligence.

Keywords: graph theory; network science; higher-order networks; hypergraphs; simplicial complexes; multilayer networks; temporal networks; higher-order dynamics; graph neural networks; hypergraph neural networks; topological data analysis; network intelligence

1. INTRODUCTION

Graph theory began with a deceptively simple modelling idea: a complicated real-world arrangement can be represented through entities and the relationships connecting them. Euler's treatment of the Königsberg bridge problem is conventionally regarded as a foundational point in this history because it abstracted geographical detail into connectivity. The significance of this abstraction is methodological as much as historical: once irrelevant physical detail is discarded, a relational system becomes amenable to formal reasoning about paths, connectivity and structure. Subsequent developments transformed graphs from objects of mathematical study into models of social, technological, biological, organizational and economic systems.

The success of the graph abstraction generated a second-order scientific development. Researchers began not only to analyze a given graph but also to ask how network structure emerges, how heterogeneous degree distributions arise, how communities form, and how processes propagate through networks. The small-world model of Watts and Strogatz (1998), the growth and preferential-attachment model of Barabási and Albert (1999), and the broader synthesis of complex network science by Newman (2003) illustrate this transition. Network science consequently became an interdisciplinary modelling framework rather than a narrow branch of discrete mathematics.

Yet the classical graph representation embeds a strong assumption: the elementary relationship is dyadic. An edge connects two nodes. This assumption is appropriate for many phenomena, but not for all. A research article written by five scientists is one collaborative event involving five people; a multidisciplinary clinical encounter may involve several professional roles and a patient simultaneously; a classroom event can expose many individuals together; and an innovation project may involve a constellation of firms, universities and public agencies. Replacing each collective event with all possible pairwise edges can preserve some connectivity information while discarding the fact that those edges originated from one group-level event.

This observation motivates the central question of the review: how and why has graph-based modelling evolved from pairwise relationships toward multilayer, temporal and higher-order representations, and what does this evolution imply for the modelling and understanding of complex systems? The paper treats the transition as an expansion of representational capacity rather than a simple replacement of one model by another. This interpretation is consistent with the higher-order literature, which emphasizes that the appropriate representation depends on the dependencies present in the empirical system and on the scientific question being asked (Battiston et al., 2020; Torres et al., 2021; Bick et al., 2023).

Three questions guide the synthesis that follows. First, what is the elementary interaction represented by the model? Second, what structural, temporal or topological information does the representation preserve? Third, what analytical and computational operations become possible — or difficult — after that representation is selected? The manuscript is organized as an evolutionary review rather than a catalogue of network types: the central analytical unit is the modelling requirement, and each major representation is introduced because it preserves a form of relational complexity that becomes important for a particular class of empirical systems. A representation is therefore not merely a data structure; it determines which relationships can be observed, measured, modelled and learned, so that changing a representation can change the downstream operator, dynamics, prediction and interpretation.

The review makes four contributions. First, it reconstructs the historical trajectory from classical graph theory to network science and higher-order modelling. Second, it compares representations using common dimensions — representation, structure, dynamics, scale, inference and interpretability. Third, it connects mathematical evolution with analytical and computational developments, including topology, signal processing and graph learning. Fourth, it proposes the EHNIF framework and a focused future research agenda.

1.1 Research questions

- RQ1. How has graph-based modelling evolved from classical graph theory to contemporary higher-order network frameworks?
- RQ2. What conceptual and analytical limitations arise when complex systems are represented exclusively through pairwise relationships?

- RQ3. How do multilayer, temporal, hypergraph and simplicial representations extend conventional graph-based modelling?
- RQ4. How does the transition toward higher-order representations alter the analysis of structure and dynamics in complex systems?
- RQ5. How are higher-order network representations being integrated with topological data analysis, signal processing and machine learning?
- RQ6. What methodological, computational and empirical challenges remain in the development of higher-order network models?

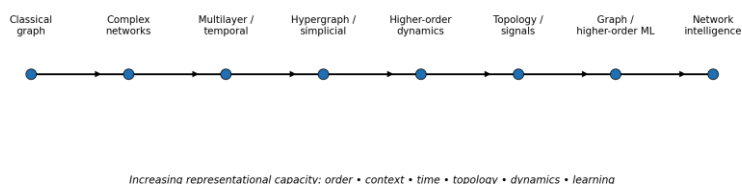


Figure 1. The review interprets graph-based modelling as an evolutionary expansion of representational capacity. The stages are not strictly linear replacements; several develop in parallel.

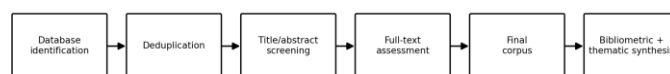
2. REVIEW METHODOLOGY

The review adopts a hybrid systematic-narrative design. This choice follows from the heterogeneity of the literature: the evidence base contains mathematical theories, formal modelling papers, algorithms, simulation studies, empirical network analyses, machine-learning methods and domain applications. A conventional effect-size meta-analysis would not be meaningful across such heterogeneous contributions. Instead, the review combines systematic retrieval, structured evidence classification, bibliometric mapping, thematic synthesis and critical conceptual integration.

2.1 Search and scope protocol

The preferred bibliographic source is Scopus, with Web of Science as a possible validation database. The search architecture uses concept blocks covering graph theory and network science; multilayer, multiplex and temporal networks; higher-order networks, hypergraphs and simplicial complexes; higher-order dynamics; and graph or hypergraph learning.

A reproducible search should record the database, search date, fields, exact query, publication years, document types, language, inclusion criteria and duplicate-removal procedure. The review also follows the logic of PRISMA 2020 for transparent reporting of identification, screening and eligibility, while recognizing that the present manuscript is not a clinical systematic review (Page et al., 2021).



Note: methodological workflow only; corpus counts are populated from the final database export, not fabricated here.

Figure 2. Reproducible evidence-synthesis workflow. No database counts are fabricated in this manuscript; actual PRISMA numbers should be populated from the final Scopus/Web of Science export.

2.2 Inclusion and exclusion criteria

Table 1. Inclusion and exclusion criteria.

Criterion	Inclusion	Exclusion
Topic	Graph/network representation, higher-order interaction, dynamics, topology or network learning	Generic AI or graph algorithms without relational-modelling relevance
Contribution	Theory, framework, method, empirical study, simulation or substantive review	Insufficiently relevant or duplicate records
Representation	Graph, multilayer, temporal, hypergraph, simplicial or related higher-order structure	Unrelated network terminology
Language	English for the core synthesis	Non-English records where full assessment is unavailable

2.3 Evidence coding framework

Each included study should be coded along common analytical dimensions. The resulting evidence matrix enables comparison across mathematically different traditions without collapsing them into a single score. The coding dimensions include publication year, domain, representation, interaction order, dynamics, methodological approach, data type, learning paradigm, topological method, application domain and contribution type.

Table 2. Evidence coding framework.

Dimension	Representative categories
Representation	Graph; multilayer; temporal; hypergraph; simplicial; tensor/cell-complex
Interaction order	Pairwise; triadic; arbitrary k-way
Dynamics	Static; temporal; adaptive; coevolutionary
Method	Analytical; simulation; empirical; algorithmic; machine learning
Data	Synthetic; observational; event-based; bibliographic; multimodal
Learning	None; embedding; GNN; HGNN; topological learning
Topology	None; homology; persistent homology; Hodge theory
Contribution	Foundational; methodological; empirical; review; theoretical critique

2.4 Bibliometric analysis protocol

Bibliometric analysis is included because the central claim is evolutionary. Publication counts by year can reveal growth; co-citation can identify intellectual foundations; bibliographic coupling can reveal research fronts; co-authorship can describe collaboration; and keyword co-occurrence can expose conceptual transitions. Bibliometrix and VOSviewer provide established tools for such analyses (Aria & Cuccurullo, 2017; van Eck & Waltman, 2010; Donthu et al., 2021).

A crucial methodological distinction is made between bibliometric protocol and bibliometric findings. Because a final Scopus or Web of Science export was not supplied with the present manuscript, this version does not invent publication counts, citation totals, countries, clusters

or thematic-map coordinates. Instead, it specifies the analysis that should be executed on the final corpus. Once the database export is available, the figures can be populated without changing the conceptual structure of the paper.

Table 3. Bibliometric analysis protocol.

Analysis	Unit	Purpose	Expected interpretation
Annual production	Year	Track growth	Periods of acceleration or stabilization
Keyword co-occurrence	Keyword	Map conceptual structure	Emergence and convergence of themes
Co-citation	Publication	Identify intellectual foundations	Foundational schools
Bibliographic coupling	Publication	Identify active research fronts	Current thematic communities
Co-authorship	Author/institution/country	Map collaboration	Research-network structure
Thematic evolution	Keyword cluster/time	Trace conceptual change	Movement across modelling paradigms

3. HISTORICAL FOUNDATIONS: FROM CONNECTIVITY TO NETWORK MODELLING

The historical evolution is best understood as a sequence of modelling requirements. Eulerian graph abstraction addressed connectivity. Classical graph theory formalized vertices, edges, paths, cycles, trees, colouring, planarity and connectivity. Random graph theory introduced uncertainty and structural emergence. Social network analysis shifted attention toward relational position and centrality. Complex network science then integrated empirical datasets with statistical-physical models of organization and dynamics.

The transition from mathematical object to empirical model is especially important. Once a graph represents a real system, the choice of what constitutes a node and what constitutes an edge becomes substantive. A social network can represent friendship, communication, advice, collaboration or influence; these are not interchangeable. Freeman's (1979) work on centrality exemplifies the shift from merely identifying connections to asking how structural position can be quantified.

Complex network science introduced a new vocabulary for empirical structure. Watts and Strogatz (1998) demonstrated that networks can combine high local clustering with short characteristic path lengths. Barabási and Albert (1999) showed how growth and preferential attachment can generate highly heterogeneous degree distributions. Fortunato (2010) synthesized community-detection methods, while Milo et al. (2002) demonstrated the usefulness of recurring network motifs. Boccaletti et al. (2006) integrated structural and dynamical perspectives.

These developments did not remove the pairwise assumption. Rather, they made it more powerful. The same edge-based language could represent many systems and support increasingly sophisticated measures and dynamics. The later recognition of higher-order interactions therefore emerges from the success of graph modelling as much as from its limitations.

4. THE PAIRWISE NETWORK PARADIGM AND ITS ANALYTICAL POWER

For an undirected simple graph $G = (V, E)$, each edge $e = \{i, j\}$ represents a dyadic relation. The adjacency matrix A has entries A_{ij} indicating whether nodes i and j are connected. Weighted, directed and signed variants add intensity, direction and polarity. From this representation arise degree, path length, clustering, centrality, community structure and a broad family of dynamical models.

The pairwise paradigm has major advantages. It is mathematically tractable, computationally efficient relative to many higher-order alternatives, supported by mature algorithms, and often highly interpretable. Many real interactions are genuinely dyadic: a direct communication channel, a road segment connecting two locations, or a bilateral transaction can naturally be encoded as an edge. Consequently, higher-order modelling should not be presented as an automatic improvement over graphs.

The limitation arises when a collective event is itself the causal or observational unit. Suppose a group event involves $\{A, B, C\}$. Its clique projection generates AB , AC and BC . The projection preserves that the three entities are mutually connected, but it does not, by itself, establish that three independent dyadic interactions occurred. This distinction matters when the outcome depends on the simultaneous presence of the whole group.

The general modelling principle is therefore: representation choice should follow the data-generating mechanism. If the empirical process produces dyads, a graph may be sufficient. If it produces group events, a hypergraph may preserve the event more faithfully. If group relations exhibit a meaningful closure structure, a simplicial complex may be appropriate. If relation types coexist, a multilayer representation may be necessary; if interactions evolve in time, temporal structure must also be retained.

5. BEYOND PAIRWISE REPRESENTATION: MULTILAYER AND TEMPORAL NETWORKS

The evolution beyond simple graphs has not followed a single path. One branch preserves multiple relational contexts through multilayer or multiplex networks; another preserves temporal order; another represents group-level interactions directly. Kivelä et al. (2014) provided a general multilayer framework, while Boccaletti et al. (2014) synthesized the structure and dynamics of multilayer systems. Holme and Saramäki (2012) established temporal networks as a major framework for time-dependent interactions, and Gross and Blasius (2008) reviewed adaptive coevolutionary networks.

A multilayer network can be represented as $M = (V_M, E_M, L)$, where L indexes layers. The same pair of entities can be connected in different layers, for example through communication, collaboration and financial relationships. Aggregating these layers into one graph may hide the fact that a relationship has multiple meanings. Temporal networks go further by representing $A_{ij}(t)$, so that the order and timing of interactions become part of the model.

Multilayer and higher-order modelling solve different problems. Multilayer modelling primarily expands relational context, whereas higher-order modelling expands interaction cardinality. They can, however, be combined. A group interaction can occur in a specific layer at a specific time, yielding a structure such as $H^{(l)}(t)$. This suggests that network complexity is multidimensional rather than one-dimensional.

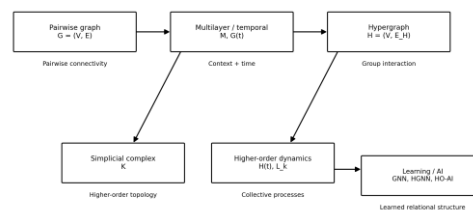


Figure 3. Representation families should be distinguished by the type of complexity they preserve. Multilayer and temporal models primarily expand context and time; hypergraphs and simplicial complexes explicitly expand interaction order and topology.

6. HYPERGRAPHS: MAKING GROUP INTERACTIONS EXPLICIT

A hypergraph $H = (V, E_H)$ generalizes the edge concept by allowing a hyperedge e to contain any number of vertices. The incidence matrix H_{ve} indicates whether vertex v belongs to hyperedge e . Hyperdegree, hyperedge cardinality and weighted incidence structures provide direct measures of participation in group-level relations (Bretto, 2013).

The representational gain is easiest to see through scientific collaboration. A paper authored by five researchers can be stored as one hyperedge containing five authors. Its clique expansion generates ten pairwise edges because $C(5,2) = 10$. The projected graph can be useful for some questions, but it cannot uniquely recover the original group event. Projection therefore constitutes a transformation with possible information loss.

Hypergraph learning extends this representation into computation. Zhou et al. (2006) developed hypergraph-based learning methods for clustering, classification and embedding. Feng et al. (2019) introduced Hypergraph Neural Networks, while Bai et al. (2021) developed hypergraph convolution and attention mechanisms. These developments illustrate the computational continuation of the representational shift: the group becomes a message-passing or aggregation unit rather than a collection of independent edges.

Hypergraph models nevertheless introduce challenges. Hyperedges must be constructed from raw observations, their semantics must be clear, and the number of possible subsets grows rapidly. Moreover, a hyperedge does not automatically imply a hierarchy of lower-order interactions. This becomes particularly important when comparing hypergraphs with simplicial complexes.

7. SIMPLICIAL COMPLEXES AND HIGHER-ORDER TOPOLOGY

A simplicial complex K is a collection of simplices satisfying the downward-closure property: if a simplex belongs to K , all of its faces also belong to K . A k -simplex contains $k + 1$ vertices. This requirement distinguishes simplicial complexes from general hypergraphs. A four-node group can be a hyperedge without requiring every subset to have the same structural status; a 3-simplex, by contrast, carries an associated collection of faces.

Simplicial complexes enable a richer topological language. Boundary operators map k -dimensional chains to $(k - 1)$ -dimensional chains. Their composition satisfies $\partial_{k-1}\partial_k = 0$, and the resulting homology groups $H_k = \ker(\partial_k)/\text{im}(\partial_{k+1})$ quantify topological structures such as connected components and higher-dimensional holes. Betti numbers $\beta_k = \dim(H_k)$ summarize these structures. Hodge Laplacians, $L_k = B_k^T B_k + B_{k+1} B_{k+1}^T$, extend Laplacian-based analysis to higher-dimensional signals (Horak & Jost, 2013; Schaub et al., 2020).

Topological data analysis (TDA) adds another dimension through persistent homology. Instead of examining a single complex, a filtration $K_{\epsilon_1} \subseteq K_{\epsilon_2} \subseteq \dots$ tracks how topological features appear and disappear as a scale parameter changes. Persistence diagrams and related summaries provide multiscale descriptors of data topology (Otter et al., 2017; Carlsson, 2020; Chazal & Michel, 2021).

This topological branch is important for the evolutionary argument because it shows that higher-order modelling is not only about adding larger edges. It changes the mathematical objects used for inference — from adjacency relations to boundary operators, homology groups and Hodge Laplacians. The resulting analysis can capture structures that are invisible to conventional node-level or edge-level statistics.

8. HIGHER-ORDER MATHEMATICAL MODELLING: TRANSFORMATIONS AND INFORMATION LOSS

Different representations can sometimes be transformed into one another, but transformation does not imply equivalence. Two common operations are clique expansion and star expansion. Clique expansion replaces a hyperedge e by pairwise edges among its members. Star expansion introduces an auxiliary interaction node connected to every participant. These transformations are computationally useful but can alter the semantics of the original data.

Consider $e = \{A, B, C, D\}$. Clique expansion creates six dyadic edges. The resulting graph answers questions about pairwise co-membership, but the original hyperedge answers a different question: which four entities participated in the same collective event? If a contagion process depends on group participation, the projected graph may produce different dynamics. Thus, computational convenience should not be confused with representational equivalence.

The modelling taxonomy can be summarized as follows: graphs encode pairwise connectivity; weighted and signed graphs add intensity or polarity; multilayer networks add relationship multiplicity; temporal networks add time; hypergraphs add arbitrary group interactions; simplicial complexes add structured higher-order topology; tensors and related algebraic objects can encode multidimensional relations; and higher-order learning models seek to learn representations from these structures.

9. HIGHER-ORDER DYNAMICS

The consequences of higher-order representation become especially visible in dynamics. A conventional network dynamical system may be written schematically as $dx_i/dt = F_i(x_i) + \sum_j A_{ij} G_{ij}(x_i, x_j)$. Higher-order interactions add terms such as $\sum_{jk} B_{ijk} H_{ijk}(x_i, x_j, x_k)$, allowing the evolution of one node to depend on a collective configuration.

Battiston et al. (2020, 2021) and Majhi et al. (2022) synthesize a growing literature on synchronization, contagion, consensus, cooperation and other collective processes. Iacopini et al. (2019) showed how simplicial interactions can produce nonlinear social-contagion mechanisms in which group-level reinforcement changes spreading behaviour. Such models demonstrate that the difference between pairwise and higher-order representations can be dynamical rather than merely descriptive.

A particularly important caution concerns the distinction between hypergraphs and simplicial complexes. Zhang, Lucas and Battiston (2023) showed that higher-order interactions can shape collective dynamics differently under hypergraph and simplicial representations. Consequently, the phrase 'higher-order network' should not be treated as a single universal

mathematical model. The topology and closure assumptions embedded in the representation can influence the predicted behaviour.

Higher-order dynamics also interact with temporal and adaptive structure. In a dynamic hypergraph $H(t)$, both membership and group size can change. Nodes may alter their state in response to group interactions while those state changes alter future group formation. This produces a structure–state feedback loop that is more complex than either a static graph or a conventional temporal edge list.

10. HIGHER-ORDER MEASURES AND NETWORK TOPOLOGY

The transition to higher-order structure requires corresponding measures. Degree and centrality remain useful but can become incomplete when a node participates in groups of different sizes or roles. Hypergraph eigenvector centralities and other higher-order centrality measures seek to capture this structure directly (Benson, 2019). Motif and spectral approaches similarly extend the vocabulary of network organization.

The topological perspective adds information not reducible to centrality. Homology distinguishes structures according to dimensions, while Hodge theory decomposes higher-order signals into gradient, curl and harmonic components. These tools permit questions such as whether a signal on edges is locally consistent, circulation-dominated or associated with topological constraints (Schaub et al., 2020, 2021).

The analytical evolution can therefore be described as a movement from scalar node statistics toward operators that act on edges, faces and higher-dimensional structures. This is not simply a quantitative increase in complexity; it changes the mathematical domain in which a signal or process is defined.

11. NETWORK SIGNAL PROCESSING AND THE EMERGENCE OF STRUCTURAL SIGNALS

Graph signal processing interprets values attached to network entities as signals whose meaning depends on topology. Higher-order network signal processing extends this principle to edges, triangles and other higher-dimensional objects. Schaub et al. (2021) describe a framework in which higher-order structures support signal processing beyond the conventional node-edge setting.

The importance of this development is conceptual. Once higher-order structures are treated as domains for signals, the network is no longer merely a static relational map. It becomes an operator on which information can be filtered, decomposed and propagated. Hodge Laplacians provide a natural mathematical basis for such operations. This connects topology, dynamics and computation within one analytical pipeline.

A mature higher-order modelling ecosystem can therefore be represented as: empirical interaction data $\rightarrow$ structural representation $\rightarrow$ higher-order operator $\rightarrow$ signal transformation $\rightarrow$ learning or inference. The operator is critical because it determines which patterns are treated as smooth, oscillatory, harmonic or otherwise meaningful.

12. FROM GRAPH NEURAL NETWORKS TO HIGHER-ORDER NETWORK LEARNING

Graph neural networks (GNNs) shifted network analysis from handcrafted measures toward representation learning. Early graph neural-network formulations (Scarselli et al., 2009) were followed by spectral and spatial developments, including graph convolutional networks (Defferrard et al., 2016; Kipf & Welling, 2017), inductive representation learning (Hamilton

et al., 2017), attention-based networks (Veličković et al., 2018) and expressive message-passing formulations (Xu et al., 2019).

Most conventional GNNs operate through pairwise message passing. A node aggregates information from its neighbours, and repeated layers expand the receptive field. This is powerful, but if the underlying phenomenon is a group interaction, the pairwise message-passing architecture may encode the group indirectly through several edges.

Hypergraph neural networks change the computational unit. Feng et al. (2019) incorporated hypergraph structure into neural representation learning. HyperGCN (Yadati et al., 2019) explored graph-convolutional learning on hypergraphs, while Jiang et al. (2019) addressed dynamic hypergraph neural networks. Bai et al. (2021) added hypergraph convolution and attention. Wang et al. (2023) connected hypergraph energy functions to neural-network learning, providing another route from higher-order structure to learned embeddings.

The central computational transition is therefore from pairwise message passing to collective message passing. This does not mean that every higher-order model is inherently superior. Rather, it creates the possibility of learning directly from group structure when group membership carries information relevant to the prediction task.

13. HIGHER-ORDER AI AND EXPLAINABILITY

The next stage of the evolutionary argument is higher-order AI: systems in which graph, hypergraph, simplicial or temporal higher-order representations become inputs to learning, prediction and explanation. The term should be used cautiously as a conceptual umbrella rather than as a claim that one unified model already exists.

A higher-order learning system can be expressed schematically as $X, H, K \rightarrow \text{higher-order operators} \rightarrow \text{representation learning} \rightarrow \text{embeddings} \rightarrow \text{prediction}$. In dynamic settings, $H(t)$ or $K(t)$ must also be learned. The challenge is to preserve relational semantics while obtaining computationally efficient representations.

Explainability becomes more demanding because an explanation may need to identify not only an important node or edge but an important group, interaction order, layer and temporal context. A useful higher-order explanation object can therefore be represented conceptually as $E = (v, e, k, t, w)$, where v is an influential entity, e an influential interaction, k the interaction order, t the context or time and w an importance weight. This is a research direction rather than an established universal standard.

Causality is another unresolved issue. A network edge or hyperedge indicates a relationship or co-occurrence, not necessarily a causal mechanism. Higher-order temporal data may improve causal identification by preserving event order, but causal inference requires explicit assumptions, interventions or suitable identification strategies. Higher-order network intelligence should therefore distinguish association, prediction, explanation and causal inference.

14. APPLICATIONS ACROSS COMPLEX SYSTEMS

Higher-order modelling is most relevant where the empirical phenomenon is collective, contextual or event-based. The application literature spans social systems, biology, ecology, epidemiology, technology, scientific collaboration and organizational systems. The 2020 higher-order-network review provides a broad cross-domain foundation for this perspective (Battiston et al., 2020).

In social systems, group decisions, team interactions and collective contagion may be represented by group events rather than independent dyads. In epidemiology, a classroom,

household, workplace or mass gathering can generate collective exposure. The modelling question is not whether pairwise contacts exist — they often do — but whether the outcome depends on the simultaneous group context.

In biology, protein complexes and multi-component molecular systems provide natural examples of interactions that may involve more than two entities. Hypergraph representations can preserve complex membership. In ecology, multiple species can participate in the same ecological interaction, making group-based representations potentially informative.

Healthcare provides a particularly promising application space. A clinical process can involve a patient, multiple professionals, a facility, a diagnosis and a treatment simultaneously. Higher-order representations may therefore be useful for modelling multidisciplinary care, treatment combinations, referral pathways and care episodes. Such use should be empirically justified rather than assumed to improve prediction.

Scientific collaboration provides an especially transparent example. A multi-author paper is naturally an event containing several authors. A hypergraph can retain the collaboration as one unit, while a projected co-authorship graph represents all pairwise co-memberships. This distinction becomes important when studying team formation, interdisciplinarity and knowledge production.

For management research, higher-order networks offer a bridge to organizational phenomena. A project can involve HR, finance, operations and IT simultaneously; an innovation ecosystem can involve firms, universities, investors, regulators and customers; and industry–institution interaction can involve multiple organizations and stakeholders within one collaborative event. The potential contribution is therefore not merely technological: higher-order network modelling can provide a formal language for collective organizational relationships.

15. COMPARATIVE SYNTHESIS OF GRAPH-BASED REPRESENTATIONS

Table 4. Comparative synthesis of graph-based representations.

Dimension	Pairwise graph	Multilayer temporal /	Hypergraph	Simplicial complex	Higher-order learning
Basic unit	Pair	Pair context/time +	Arbitrary set	Structured simplex	Learned relational structure
Primary complexity preserved	Connectivity	Context and time	Group membership	Higher-order topology	Learned collective patterns
Interaction order	2	Usually 2	$k \geq 2$	$k \geq 2$ with closure	Task-dependent
Group interaction	Indirect	Indirect	Direct	Direct closure +	Direct/learned
Topology	Graph topology	Layered topology	Incidence structure	Rich higher-dimensional topology	Depends on architecture
Dynamics	Mature	Mature/developing	Developing	Developing	Rapidly developing

Dimension	Pairwise graph	Multilayer temporal /	Hypergraph	Simplicial complex	Higher-order learning
Interpretability	Generally high	Moderate	Moderate/complex	Complex	Challenging
Computational burden	Relatively low	Moderate	Higher	Higher	Potentially high
Main risk	Over-simplification when relations are collective	Aggregation across layers/time	Ambiguous hyperedge semantics	Over-restrictive closure assumptions	Black-box or representation mismatch

The comparison suggests that there is no single evolutionary endpoint. Each representation preserves a different dimension of complexity. A graph may be preferable when dyadic relations are the true empirical unit; multilayer and temporal networks are needed when context or timing matters; hypergraphs are suitable when arbitrary group membership is primary; simplicial complexes are useful when higher-order interactions have meaningful closure and topological structure; and higher-order learning becomes relevant when these representations must support prediction or representation learning.

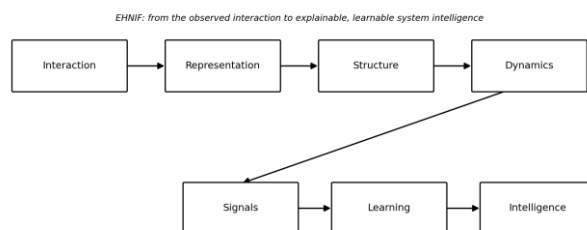


Figure 4. *The EHNIF framework integrates the representational, analytical and computational stages of the field.*

16. THE EVOLUTIONARY HIGHER-ORDER NETWORK INTELLIGENCE FRAMEWORK (EHNIF)

The principal conceptual contribution of this review is the Evolutionary Higher-Order Network Intelligence Framework (EHNIF). It synthesizes the literature around seven linked stages: Interaction → Representation → Structure → Dynamics → Signals → Learning → Intelligence.

Stage 1, Interaction, asks what the empirical event actually is. Stage 2, Representation, determines whether a graph, multilayer network, temporal graph, hypergraph, simplicial complex or hybrid representation is appropriate. Stage 3, Structure, identifies connectivity, topology and organization. Stage 4, Dynamics, models how states change over the structure. Stage 5, Signals, treats information as a structured quantity operating on nodes, edges or higher-dimensional elements. Stage 6, Learning, uses embeddings, GNNs, HGNNs or topological learning. Stage 7, Intelligence, concerns prediction, explanation, causal reasoning and decision support.

The framework yields three simultaneous evolutionary dimensions. The mathematical evolution is $G \rightarrow M \rightarrow H \rightarrow K$. The analytical evolution is metrics → topology → dynamics

→ signals. The computational evolution is algorithms → GNNs → HGNNs → higher-order AI. These dimensions intersect rather than replace one another.

A central proposition follows: a representation is not merely a storage format. It determines which relationships are observable, measurable, modelable and learnable. Consequently, a change from H to a pairwise projection G can change downstream operators, dynamics and learned representations. The review therefore treats representation selection as part of scientific inference.

17. CRITICAL RESEARCH GAPS

The literature supports a substantial research agenda, but the gaps are best expressed as specific methodological problems rather than generic calls for 'more research'.

17.1 Representation selection

There is no universal rule for selecting among graph, multilayer, temporal, hypergraph and simplicial representations. A rigorous representation-selection framework should consider the data-generating process, interaction semantics, temporal structure, research question and computational constraints. Torres et al. (2021) provide an important foundation for this perspective.

17.2 Empirical ground truth

A considerable share of higher-order research uses synthetic networks and simulations. These are valuable for mechanism discovery, but empirical benchmarks are necessary to determine when higher-order structure improves inference in real systems. Benchmark datasets should retain raw interaction events, group membership, time stamps and outcomes wherever possible.

17.3 Hypergraph–simplicial ambiguity

Hypergraphs and simplicial complexes should not be treated as synonyms. The downward-closure property of simplicial complexes encodes assumptions that are absent from general hypergraphs. Comparative studies such as Zhang et al. (2023) show that this distinction can affect collective dynamics.

17.4 Dynamic higher-order networks

Static $H = (V, E)$ models are insufficient for systems in which group membership changes. Dynamic higher-order systems require models of $H(t)$, potentially with $V(t)$ and $E(t)$, while accounting for feedback between node states and topology. This is a major opportunity for adaptive higher-order modelling.

17.5 Scalability

The number of possible k -node subsets is $C(n, k)$, while the number of all non-empty subsets is $2^n - 1$. Although real systems are sparse, higher-order representations can still become computationally demanding. Sparse algorithms, sampling, approximation, distributed computation and hardware-aware learning are therefore important.

17.6 Explainability and causal inference

Higher-order models must explain group-level contributions, not only node importance. Causal inference is harder still because association within a hyperedge does not imply causal influence. Future work should integrate temporal higher-order representations with explicit causal assumptions.

17.7 Benchmarking and reproducibility

Fair evaluation requires the same data and prediction task to be used across pairwise and higher-order baselines. Benchmarking should report predictive performance together with computational cost, robustness, scalability, information preservation and interpretability. This avoids the assumption that greater representational complexity necessarily produces better results.

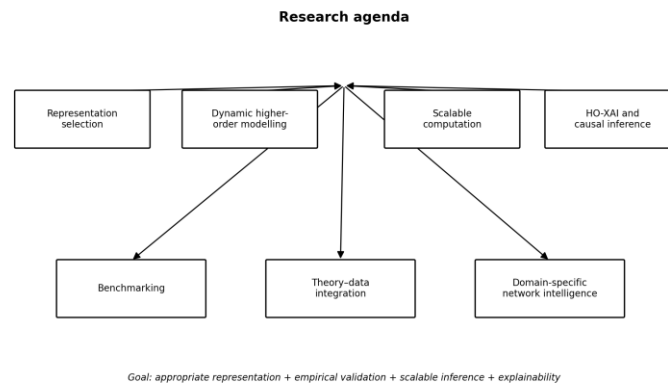


Figure 5. Priority research directions emerging from the critical synthesis. The framework emphasizes methodological fit and empirical validation rather than complexity for its own SAKE.

18. FUTURE RESEARCH AGENDA

The next generation of research should move from 'more complex networks' toward more appropriate and empirically justified representations. Seven directions are especially important.

- Representation intelligence: develop data-driven methods for selecting G , M , $G(t)$, H or K according to interaction semantics and inferential objectives.
- Dynamic higher-order modelling: jointly learn evolving topology and node or group states through $H(t)$ and adaptive higher-order processes.
- Higher-order explainable AI: create explanation methods that identify important groups, interaction orders, layers and temporal contexts.
- Higher-order causal inference: combine event-level temporal information with causal discovery and identification strategies.
- Scalable computation: develop sparse, approximate and distributed algorithms for large hypergraphs and simplicial complexes.
- Empirical benchmarking: establish open datasets and common evaluation protocols comparing pairwise and higher-order models on identical tasks.
- Domain-specific network intelligence: extend higher-order methods to healthcare, biology, organizational systems, innovation ecosystems and sustainability.

An especially promising cross-disciplinary direction is the integration of higher-order network modelling with organizational and management research. Collective work is often organized through teams, projects, stakeholder constellations and institutional collaborations. These phenomena can naturally be represented as group-level events. A rigorous

management research program could test whether preserving these higher-order structures changes explanatory or predictive conclusions relative to conventional network projections.

19. DISCUSSION

The evolutionary synthesis suggests that graph-based modelling has not followed a simple path from 'simple' to 'complex'. Instead, it has expanded along several dimensions. Interaction order increased from pairs to groups; relational context expanded from one layer to multiple layers; time was incorporated explicitly; topology extended analysis beyond connectivity; and machine learning introduced learned representations. Higher-order network science is therefore better understood as a convergence of multiple evolutionary trajectories.

This interpretation also changes how the literature should be evaluated. The central question is not whether a hypergraph contains more information than a graph in an abstract sense. The relevant question is whether the additional structure corresponds to real dependencies that matter for the scientific task. If it does, projecting to pairwise edges can discard information. If it does not, higher-order modelling may add unnecessary complexity.

The review also highlights a methodological asymmetry. It is relatively easy to construct a higher-order representation from pairwise data by imposing group structures, but much harder to establish empirically that a group interaction is the correct data-generating unit. Consequently, future research should emphasize event-level data collection and validation, not only new mathematical operators.

Finally, the transition to network intelligence should be treated as an extension of scientific modelling rather than a technological endpoint. Prediction without representation validity can amplify modelling errors. Explainability without correct relational semantics can produce plausible but misleading narratives. The EHNIF framework therefore places interaction and representation before learning and intelligence.

20. LIMITATIONS OF THE REVIEW

This manuscript has several limitations. First, the bibliometric section specifies a reproducible analysis protocol but does not report fabricated database counts because a final Scopus/Web of Science export was not supplied. Second, the literature is heterogeneous, so the review uses structured thematic synthesis rather than quantitative meta-analysis. Third, applications are discussed conceptually across domains, and domain-specific empirical validation remains a separate research task.

These limitations are methodological boundaries rather than defects to be hidden. Making them explicit strengthens reproducibility and prevents the review from presenting proposed bibliometric outputs as observed findings.

CONCLUSION

Graph-based modelling has evolved from a mathematical language for pairwise connectivity into a broad framework for representing, analysing and learning from complex relational systems. The evolution was driven by recurring modelling requirements: uncertainty encouraged random graphs; empirical heterogeneity stimulated complex network science; multiple relationship types motivated multilayer models; changing interactions motivated temporal and adaptive networks; group interactions motivated hypergraphs; structured higher-order relations motivated simplicial complexes; and the need for scalable inference stimulated graph and higher-order machine learning.

The central conclusion is therefore not that graphs have been replaced by hypergraphs. Rather, the field has progressively expanded the meaning of a relationship. A relationship can

be a pair, a layered pair, a time-ordered event, a group, a higher-dimensional topological structure or a learned collective representation. Each modelling choice determines what can subsequently be measured, simulated, learned and inferred.

The EHNIF framework captures this progression as Interaction → Representation → Structure → Dynamics → Signals → Learning → Intelligence. Its principal implication is methodological: the route to network intelligence should begin with an empirically defensible representation of interaction. Future progress will depend less on adding complexity for its own sake and more on developing models that are faithful to event semantics, empirically validated, computationally scalable, interoperable and explainable.

The review consequently positions higher-order network science as a continuing evolution of graph-based modelling rather than a departure from graph theory. Its long-term significance lies in the possibility of connecting mathematically rigorous representations with real-world collective systems and, ultimately, with transparent forms of network-based scientific inference and decision support.

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